Math 10A
Worksheet, Discussion #20; **Friday, 7/13/2018**
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1 Error Bounds

1.1 Concepts

1. The formula for the errors of integrating $\int_a^b f(x)dx$ are:

$$E_L = E_R = \frac{K_1(b-a)^2}{2n}, \quad E_T = \frac{K_2(b-a)^3}{12n^2}, \quad E_M = \frac{K_2(b-a)^3}{24n^2}, \quad E_S = \frac{K_4(b-a)^5}{180n^4},$$

where K_i is the maximum $|f^(i)(x)|$, the ith derivative of f, is on the interval $[a, b]$.

1.2 Problems

2. True False For calculating the error bound when using left endpoint method when approximating the integral of f on the interval $[a, b]$, we use $K_1 = f'(a)$.

3. True False The error bound gives us what the exact error of using the different approximation techniques are.

4. True False If the second derivative is negative, then the Trapezoid rule and midpoint rule both underestimate the true area.

5. True False If the first derivative is positive, then the left endpoint and right endpoint method both underestimate the true area.

6. How many intervals do we need to use to approximate $\displaystyle\int_1^2 x^2 dx$ within $0.001 = 10^{-3}$ using the midpoint rule? Trapezoid rule? Simpson's rule?

7. How many intervals do we need to use to approximate $\displaystyle\int_0^1 \cos(2x)dx$ within $0.001 = 10^{-3}$ using Simpson's rule?

8. How many intervals do we need to use to approximate $\displaystyle\int_0^2 e^{2x} dx$ within $0.001 = 10^{-3}$ using Simpson's rule?

9. How many intervals do we need to use to approximate $\displaystyle\int_{-1}^1 x^3 dx$ within $0.001 = 10^{-3}$ using Simpson's rule?

10. How many intervals do we need to use to approximate $\int_1^3 \ln x dx$ within $0.001 = 10^{-3}$ using Simpson's rule?

11. How many intervals do we need to use to approximate $\int_1^2 xe^x dx$ within $0.001 = 10^{-3}$ using Simpson's rule?

12. How many intervals do we need to use to approximate $\int_1^4 \sqrt{x} dx$ within $0.001 = 10^{-3}$ using Simpson's rule?